

2007 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)

calculator permitted

3. The wind chill is the temperature, in degrees Fahrenheit ($^{\circ}\text{F}$), a human feels based on the air temperature, in degrees Fahrenheit, and the wind velocity v , in miles per hour (mph). If the air temperature is 32°F , then the wind chill is given by $W(v) = 55.6 - 22.1v^{0.16}$ and is valid for $5 \leq v \leq 60$.
- (a) Find $W'(20)$. Using correct units, explain the meaning of $W'(20)$ in terms of the wind chill.
- (b) Find the average rate of change of W over the interval $5 \leq v \leq 60$. Find the value of v at which the instantaneous rate of change of W is equal to the average rate of change of W over the interval $5 \leq v \leq 60$.
- (c) Over the time interval $0 \leq t \leq 4$ hours, the air temperature is a constant 32°F . At time $t = 0$, the wind velocity is $v = 20$ mph. If the wind velocity increases at a constant rate of 5 mph per hour, what is the rate of change of the wind chill with respect to time at $t = 3$ hours? Indicate units of measure.
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WRITE ALL WORK IN THE EXAM BOOKLET.

END OF PART A OF SECTION II

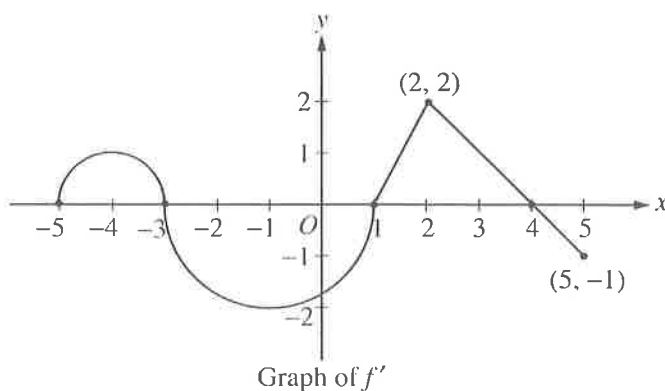
2007 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)

CALCULUS AB
SECTION II, Part B

Time—45 minutes

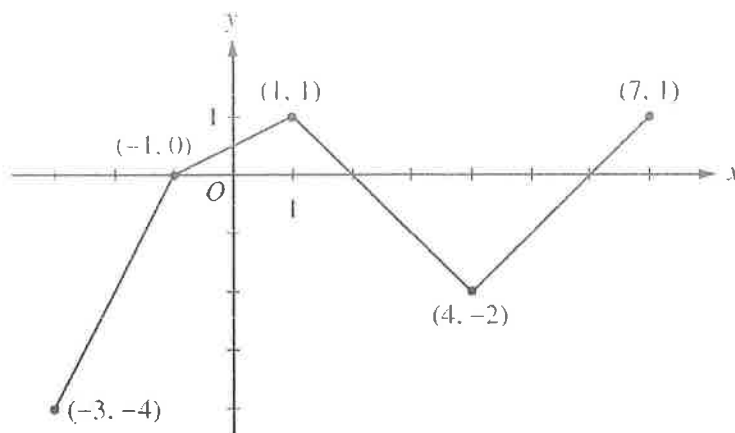
Number of problems—3

No calculator is allowed for these problems.



4. Let f be a function defined on the closed interval $-5 \leq x \leq 5$ with $f(1) = 3$. The graph of f' , the derivative of f , consists of two semicircles and two line segments, as shown above.
- For $-5 < x < 5$, find all values x at which f has a relative maximum. Justify your answer.
 - For $-5 < x < 5$, find all values x at which the graph of f has a point of inflection. Justify your answer.
 - Find all intervals on which the graph of f is concave up and also has positive slope. Explain your reasoning.
 - Find the absolute minimum value of $f(x)$ over the closed interval $-5 \leq x \leq 5$. Explain your reasoning.

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No
calculatorGraph of g'

5. Let g be a continuous function with $g(2) = 5$. The graph of the piecewise-linear function g' , the derivative of g , is shown above for $-3 \leq x \leq 7$.
- Find the x -coordinate of all points of inflection of the graph of $y = g(x)$ for $-3 < x < 7$. Justify your answer.
 - Find the absolute maximum value of g on the interval $-3 \leq x \leq 7$. Justify your answer.
 - Find the average rate of change of $g(x)$ on the interval $-3 \leq x \leq 7$.
 - Find the average rate of change of $g'(x)$ on the interval $-3 \leq x \leq 7$. Does the Mean Value Theorem applied on the interval $-3 \leq x \leq 7$ guarantee a value of c , for $-3 < c < 7$, such that $g''(c)$ is equal to this average rate of change? Why or why not?

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2008 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)

CALCULUS AB
SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.

1. Let R be the region in the first quadrant bounded by the graphs of $y = \sqrt{x}$ and $y = \frac{x}{3}$.

- (a) Find the area of R .
 - (b) Find the volume of the solid generated when R is rotated about the vertical line $x = -1$.
 - (c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the y -axis are squares. Find the volume of this solid.
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Calculator permitted

2008 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)

| | | | | | |
|---------------------------------------|---|---|----|----|----|
| Distance from the river's edge (feet) | 0 | 8 | 14 | 22 | 24 |
| Depth of the water (feet) | 0 | 7 | 8 | 2 | 0 |

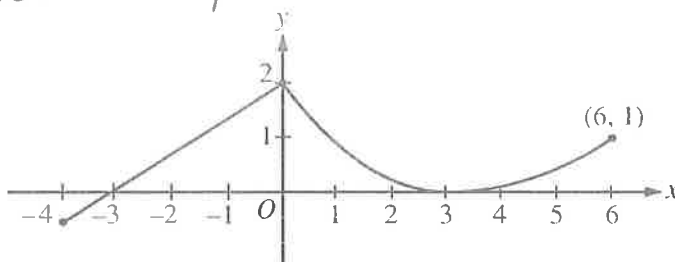
3. A scientist measures the depth of the Doe River at Picnic Point. The river is 24 feet wide at this location. The measurements are taken in a straight line perpendicular to the edge of the river. The data are shown in the table above. The velocity of the water at Picnic Point, in feet per minute, is modeled by $v(t) = 16 + 2\sin(\sqrt{t+10})$ for $0 \leq t \leq 120$ minutes.
- (a) Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate the area of the cross section of the river at Picnic Point, in square feet. Show the computations that lead to your answer.
 - (b) The volumetric flow at a location along the river is the product of the cross-sectional area and the velocity of the water at that location. Use your approximation from part (a) to estimate the average value of the volumetric flow at Picnic Point, in cubic feet per minute, from $t = 0$ to $t = 120$ minutes.
 - (c) The scientist proposes the function f , given by $f(x) = 8\sin\left(\frac{\pi x}{24}\right)$, as a model for the depth of the water, in feet, at Picnic Point x feet from the river's edge. Find the area of the cross section of the river at Picnic Point based on this model.
 - (d) Recall that the volumetric flow is the product of the cross-sectional area and the velocity of the water at a location. To prevent flooding, water must be diverted if the average value of the volumetric flow at Picnic Point exceeds 2100 cubic feet per minute for a 20-minute period. Using your answer from part (c), find the average value of the volumetric flow during the time interval $40 \leq t \leq 60$ minutes. Does this value indicate that the water must be diverted?

WRITE ALL WORK IN THE EXAM BOOKLET.

END OF PART A OF SECTION II

2009 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)

Calculator permitted



Graph of f

3. A continuous function f is defined on the closed interval $-4 \leq x \leq 6$. The graph of f consists of a line segment and a curve that is tangent to the x -axis at $x = 3$, as shown in the figure above. On the interval $0 < x < 6$, the function f is twice differentiable, with $f''(x) > 0$.
- Is f differentiable at $x = 0$? Use the definition of the derivative with one-sided limits to justify your answer.
 - For how many values of a , $-4 \leq a < 6$, is the average rate of change of f on the interval $[a, 6]$ equal to 0? Give a reason for your answer.
 - Is there a value of a , $-4 \leq a < 6$, for which the Mean Value Theorem, applied to the interval $[a, 6]$, guarantees a value c , $a < c < 6$, at which $f'(c) = \frac{1}{3}$? Justify your answer.
 - The function g is defined by $g(x) = \int_0^x f(t) dt$ for $-4 \leq x \leq 6$. On what intervals contained in $[-4, 6]$ is the graph of g concave up? Explain your reasoning.

WRITE ALL WORK IN THE EXAM BOOKLET.

END OF PART A OF SECTION II

No calculator

| | | | | | | |
|-------------------------------|---|---|-----|----|----|----|
| t (seconds) | 0 | 8 | 20 | 25 | 32 | 40 |
| $v(t)$ (meters per second) | 3 | 5 | -10 | -8 | -4 | 7 |

6. The velocity of a particle moving along the x -axis is modeled by a differentiable function v , where the position x is measured in meters, and time t is measured in seconds. Selected values of $v(t)$ are given in the table above. The particle is at position $x = 7$ meters when $t = 0$ seconds.
- Estimate the acceleration of the particle at $t = 36$ seconds. Show the computations that lead to your answer. Indicate units of measure.
 - Using correct units, explain the meaning of $\int_{20}^{40} v(t) dt$ in the context of this problem. Use a trapezoidal sum with the three subintervals indicated by the data in the table to approximate $\int_{20}^{40} v(t) dt$.
 - For $0 \leq t \leq 40$, must the particle change direction in any of the subintervals indicated by the data in the table? If so, identify the subintervals and explain your reasoning. If not, explain why not.
 - Suppose that the acceleration of the particle is positive for $0 < t < 8$ seconds. Explain why the position of the particle at $t = 8$ seconds must be greater than $x = 30$ meters.

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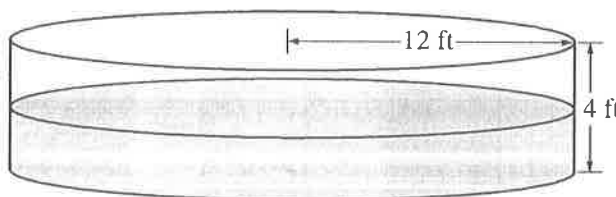
END OF EXAM

2010 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)

- ~~X~~ The function g is defined for $x > 0$ with $g(1) = 2$, $g'(x) = \sin\left(x + \frac{1}{x}\right)$, and $g''(x) = \left(1 - \frac{1}{x^2}\right)\cos\left(x + \frac{1}{x}\right)$.
- Find all values of x in the interval $0.12 \leq x \leq 1$ at which the graph of g has a horizontal tangent line.
 - On what subintervals of $(0.12, 1)$, if any, is the graph of g concave down? Justify your answer.
 - Write an equation for the line tangent to the graph of g at $x = 0.3$.
 - Does the line tangent to the graph of g at $x = 0.3$ lie above or below the graph of g for $0.3 < x < 1$? Why?

Calculator permitted

| | | | | | | | |
|--------|---|----|----|----|----|----|----|
| t | 0 | 2 | 4 | 6 | 8 | 10 | 12 |
| $P(t)$ | 0 | 46 | 53 | 57 | 60 | 62 | 63 |



3. The figure above shows an aboveground swimming pool in the shape of a cylinder with a radius of 12 feet and a height of 4 feet. The pool contains 1000 cubic feet of water at time $t = 0$. During the time interval $0 \leq t \leq 12$ hours, water is pumped into the pool at the rate $P(t)$ cubic feet per hour. The table above gives values of $P(t)$ for selected values of t . During the same time interval, water is leaking from the pool at the rate $R(t)$ cubic feet per hour, where $R(t) = 25e^{-0.05t}$. (Note: The volume V of a cylinder with radius r and height h is given by $V = \pi r^2 h$.)
- Use a midpoint Riemann sum with three subintervals of equal length to approximate the total amount of water that was pumped into the pool during the time interval $0 \leq t \leq 12$ hours. Show the computations that lead to your answer.
 - Calculate the total amount of water that leaked out of the pool during the time interval $0 \leq t \leq 12$ hours.
 - Use the results from parts (a) and (b) to approximate the volume of water in the pool at time $t = 12$ hours. Round your answer to the nearest cubic foot.
 - Find the rate at which the volume of water in the pool is increasing at time $t = 8$ hours. How fast is the water level in the pool rising at $t = 8$ hours? Indicate units of measure in both answers.

WRITE ALL WORK IN THE EXAM BOOKLET.

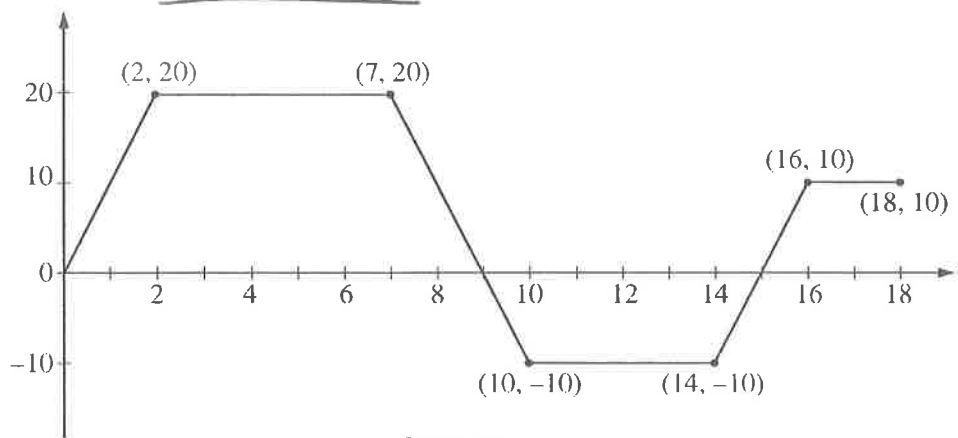
END OF PART A OF SECTION II

CALCULUS AB
SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



4. A squirrel starts at building A at time $t = 0$ and travels along a straight, horizontal wire connected to building B . For $0 \leq t \leq 18$, the squirrel's velocity is modeled by the piecewise-linear function defined by the graph above.
- At what times in the interval $0 < t < 18$, if any, does the squirrel change direction? Give a reason for your answer.
 - At what time in the interval $0 \leq t \leq 18$ is the squirrel farthest from building A ? How far from building A is the squirrel at that time?
 - Find the total distance the squirrel travels during the time interval $0 \leq t \leq 18$.
 - Write expressions for the squirrel's acceleration $a(t)$, velocity $v(t)$, and distance $x(t)$ from building A that are valid for the time interval $7 < t < 10$.

WRITE ALL WORK IN THE EXAM BOOKLET.